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UNIT 516: MECHANICS OF SOLIDS

Lesson 1: Revision of basic concepts		**Suggested Teaching Time:** 1 hour approx.
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Revision of basic concepts	• Although there is a certain level of prerequisite knowledge required for entering this course the tutor will benefit from taking time at the beginning of the course to reinforce the basic equations. • Whole-class teaching: Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms: ○ Direct Stress $\sigma = \frac{F}{A}$ ○ Direct Strain $\varepsilon = \frac{\Delta L}{L}$ ○ Young's modulus {E}; $E = \frac{\sigma\left(stress\ \sigma=\frac{F}{A}\right)}{\varepsilon\left(Strain\ \varepsilon=\frac{\Delta L}{L}\right)}$ ○ Shear modulus{G}; $G = \frac{E}{2(l+v)}$ • Tutor should demonstrate the solution of the different types of equations and then get the students to solve example questions, tutor to assist individual students, and correcting errors as required, where possible include practical elements, tutor to circulate and correct as required. • Develop Discussion to include three dimensional stress and strain including: ○ Bulk modulus {K}: $K = \frac{\sigma}{\varepsilon_v}$ where ε_v = change in volume/original volume	**Books:** http://www-mdp.eng.cam.ac.uk/web/library/enginfo/cueddatabooks/materials.pdf **Website:** http://www.freestudy.co.uk/d209/t8.pdf **Practical equipment::** laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** computer-based finite element analysis software

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UNIT 516: MECHANICS OF SOLIDS

Lesson 2: Relationship between Poisson's ratio and primary elastic modulus		**Suggested Teaching Time: 1** hour approx.
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Elastic Materials and the relationship between Poisons ratio and primary elastic modulus (A.C. 1.1 & 1.2)	• Develop concepts to explain the relationship between Poisson's ratio and primary elastic modulus: ○ explain the significance of Poisson's ratio to different types of materials including ▪ Isotropic Material: the elastic constants are the same in all directions – most metals with no pronounced grain structure. ▪ Orthotropic Material: the elastic constants have different values in the x, y and z directions –materials with grain structures such as wood or rolled metals. • Non-Isotropic Material: the elastic constants are unpredictable and the results from any two tests are never the same. This applies to materials such as glass and other ceramics under Tension and Compression.	**Books:** As per lesson 1 **Website:** http://www.freestudy.co.uk/d209/t9.pdf **Practical equipment::** laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** computer-based finite element analysis software

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Lesson 3: Complex stress strain relationships		**Suggested Teaching Time: 2** hours approx.
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Complex stress strain relationships (A.C. 1.3, 1.4 & 1.10)	• Whole-class teaching: Tutor to get the whole class involved in learner research and activity: ○ Develop the ideas previously learned to cover the real world where things do not always happen in perfect balance and symmetry and introduce the concept of stress elements in complex stress cases ○ Remember if the object is in equilibrium all forces must balance • Tutor should demonstrate the solution of the different types of equations and then get the students to solve example questions. Tutor to assist individual students, correcting errors as required – where possible including practical elements. Tutor to circulate and correct as required. ○ Introduce concept of Principle Stresses and Principal Planes • Tutor should demonstrate the solution of the different types of equations and then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. ○ Tutor-led discussion to show how the solutions to the previous examples can be solved graphically by explaining the application of Mohr's circle to combined loading examples • Tutor should demonstrate the solution of one of the previously used equations using the Mohr's circle method and then get the students to solve other questions using the same method. • Tutor should demonstrate how to determine the principle stresses for complex loading using Mohr's circle and then get the students to solve example questions.	**Books:** As per lesson 1 **Website:** http://www.freestudy.co.uk/d209/t9.pdf **Practical equipment:** laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** Computer-based finite element analysis software

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Lesson 4: Computer Modelling Part 1		**Suggested Teaching Time: 3** hours approx.
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Introduction to computer modelling (A.C. 1.9)	• Tutor-led discussion. Tutor to get the students to discuss how electronics have been developed over the years to make life easier for the engineer, progressing from early calculators, through scientific and programmable calculators to computer software. • Tutor to introduce the students to the modelling software of choice and introduce them to the basic operations and functionalities of the software. Note: Autodesk have a great range of software and tutor support and students can download free educational copies of software to use at home. • Tutor should demonstrate the solution of the some of the different types of equations previously covered using the software and then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible include practical elements. • Tutor to reinforce the fact that students need to understand the outputs and have an understanding of the formula involved so that they can see if results are not within practical realms, i.e. if there has been an error in the data entry or programming somewhere along the line.	**Books:** Robert D. Cook Finite Element Modeling for Stress Analysis ISBN-13: 978-0471107743 ISBN-10: 0471107743 **Website:** http://www.freestudy.co.uk/d209/t9.pdf **Practical equipment:** Laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** Computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/home

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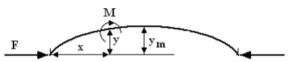
Lesson 5: Measurement of forces	**Suggested Teaching Time: 1** hour approx.	
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Physical measurement of forces (A.C. 1.5)	• Tutor-led group learning to be carried out in a laboratory environment with access to stress and strain measuring equipment. Tutor to demonstrate different measuring devices and relate to calculations covered so far and explain the operating principle of a strain gauge. Explain how it is not possible to measure shear strain so we need three measurements of direct strain to construct a circle (strain gauge rosette – two types 45^{o} and 60^{o}). • Tutor to describe how to construct a circle of stress.	**Website:** **Practical equipment:** Materials laboratory with stress and strain measuring equipment

UNIT 516: MECHANICS OF SOLIDS

<table>
<tr><td colspan="2">Lesson 6: Struts under compression</td><td>Suggested Teaching Time: 4 hours approx.</td></tr>
<tr><td colspan="3">Learning Outcome 1: Understand the behaviour of solids under elastic loading</td></tr>
<tr><th>Topic</th><th>Suggested Teaching</th><th>Suggested Resources</th></tr>
<tr><td>Euler’s theory of collapse
(A.C. 1.6 & 1.11)</td><td>• Tutor-led discussion to discuss the use of support beams in a structure and relate this to Euler’s strut theory, showing how the column would tend to buckle rather than collapse.
• Tutor to demonstrate how and where the forces are created, and build up the equations using a practical demonstration where possible. Tutor to use examples of different scenarios (Pin conditions, free conditions, built-in end conditions).
$F = n^2\pi^2\frac{EI}{L^2}$ and the corresponding deflection is y=B sin(cL)
• N.B. This theory takes no account of the compressive stress. For a metal with a compressive strength of less than 300 N/mm2 and a Young's Modulus of about 200 kN/mm2. The strut will tend to fail in compression if the slenderness ratio (Le/ k) is less than 80. Therefore, for steel Euler’s equation is not reliable for slenderness ratios less than 80 and really should not be used for slenderness ratios less than 1:20.

• Tutor should demonstrate how to determine the conditions for the stability of a strut using Euler’s theorem and then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required; examples to include: circular, solid, tubular cross section.</td><td>Books:
Robert D. Cook Finite Element Modelling for Stress Analysis ISBN-13: 978-0471107743 ISBN-10: 0471107743

Website:
http://www.freestudy.co.uk/d209/t9.pdf

Practical equipment:
laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory

Software:
computer-based finite element analysis software
E.g. Autodesk
http://www.autodesk.com/education/home</td></tr>
</table>

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UNIT 516: MECHANICS OF SOLIDS

Lesson 7 Hoop Stress	**Suggested Teaching Time: 6** hours approx.	
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Calculate the tensile hoop stress in thin rotating rings and cylinders at constant angular velocity (A.C.1.7 & 1.8)	• Tutor-led whole-class teaching: tutor to discuss a scenario concerning the stresses in a rotating mass and bring in the equations required to calculate the tensile hoop stress in thin rotating rings and cylinders at constant angular velocity. • Stress in a rotation ring can be expressed as $\sigma_z = \omega^2 \rho \,(r_1^2 + r_1 r_2 + r_2^2) / 3$ *where* r_1 = *outer radius of ring (m)* r_2 = *inner radius of ring (m)* • Tutor should demonstrate the solution of equations involving hoop stresses and then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible include practical elements. • Tutor-led whole-class teaching. The tutor should develop the hoop stress formula and apply it to a rotating disk. Stress in a rotating disc can be expressed as: σz = ω2 r2 ρ / 3 = v2 ρ / 3 where σz = stress (N/m2) ω = angular velocity (rad/s) r = radius of disc (m) ρ = density (kg/m3)	**Books:** Robert D. Cook Finite Element Modelling for Stress Analysis ISBN-13: 978-0471107743 ISBN-10: 0471107743 **Website:** http://www.freestudy.co.uk/d209/t9.pdf **Practical equipment:** laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/home

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Lesson 8: Computer Modelling Part 2	**Suggested Teaching Time: 1** hour approx.	
Learning Outcome 1: **Understand the behaviour of solids under elastic loading**		
Topic	**Suggested Teaching**	**Suggested Resources**
Further exploration of computer modelling (A.C. 1.9)	• Tutor-led discussion on the further application to solving the latest equations discussed. • Tutor should demonstrate the solution of some of the different types of equations previously covered using the software and then get students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible include practical elements. • Tutor to reinforce the fact that students need to understand the outputs and have an understanding of the formula involved so that they can see if results are not within practical realms, i.e. if there has been an error in the data entry or programming somewhere along the line.	**Books:** Robert D. Cook Finite Element Modelling for Stress Analysis ISBN-13: 978-0471107743 ISBN-10: 0471107743 **Website:** http://www.freestudy.co.uk/d209/t9.pdf **Practical equipment:** laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/home

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UNIT 516: MECHANICS OF SOLIDS

Lesson 9: Standard Beams		**Suggested Teaching Time: 1** hour approx.
Learning Outcome 2: Understand the behaviour of elastically loaded structures		
Topic	**Suggested Teaching**	**Suggested Resources**
Determination of beam reactions - revision	• Principle of moments. The basic principles should be demonstrated using simple equipment, and learners should be given the opportunity to test the principles practically. Whole-class teaching should reinforce the practical work to embed the concept that a moment of a force is the product of the magnitude of that force and the **perpendicular** distance between the turning point and the **line** of action of the force. Tutors must stress that it is **not** the actual distance between the **point** at which the force acts and the turning point that matters and, as above, this should be demonstrated practically. Learners should then be introduced to the 'principle of moments' – that clockwise moments and anti-clockwise moments are equal for co-planar systems in equilibrium. ($\Sigma M = 0$). This too should be demonstrated practically and/or checked practically by learners in small groups. • Total load = total reaction. A tutor-led discussion should be used to extend the concept of static equilibrium to the realisation that, for systems in equilibrium, not only does $\Sigma M = 0$, but ΣV and ΣH also equal zero. That $\Sigma V = 0$ implies that the algebraic sum of all vertical forces equals zero, and that total loads equal total reactions in all cases. The discussion should then be directed towards an understanding of how this concept, together with the principle of moments, can be used to determine the value of reactions for loaded beams.	**Books:** Hulse, R., Cain, J., Structural Mechanics ISBN: 0333804570 Hulse, R., Cain, J., Structural Mechanics (Worked Examples) ISBN: 0230579817 **Practical equipment** Beams, rules, hanging weights, pulleys, string **Software:** Goya Siemens PLM RISA Technology **Website:** www.istructe.org

UNIT 516: MECHANICS OF SOLIDS

Lesson 10: Standard Beams continued		**Suggested Teaching Time: 1** hour approx.
Learning Outcome 1: Understand the behaviour of solids under elastic loading		
Topic	**Suggested Teaching**	**Suggested Resources**
Determination of beam reactions - revision	• General theory of bending. Learners should be able to use the formula to design simple beams. Tutors may derive the formula from first principles but learners are not required to do so. A simple hand-out will suffice. What is important is that learners understand the importance of the variables M, I, f and y, and of using consistent units. • First and second moments of area. Tutors must emphasise the importance of sectional shape in beam sizing. Learners must be aware of the various formulae required to determine the second moment of area (I) - also known as the 'moment of inertia' - although practice calculations should be restricted to beams of rectangular section for timber and concrete and universal beam sections for steel. Learners must be given the opportunity to determine moments of inertia by using the formulae and by extraction of the values from tables, once the section modulus (z) has been determined. • The class should be divided into several small groups and each should be given similar data to allow them to determine the required size of a beam. Comparing the answers will show that there are several beam sizes that satisfy the requirements for a given loading condition. • Tutor-led discussion about the effect of differences in breadth, depth and sectional shape should lead to an agreed conclusion concerning the most practical size of beam to be used, and why this is so.	**Books:** Hulse, R., Cain, J., *Structural Mechanics* ISBN: 0333804570 **Manuals** Steel Designers' Manual (SC!) Code of Practice for Structural Use of Concrete 2013 BS 5268-2:2002 Structural Use of Timber: Part 2 **Software:** Goya Siemens PLM **Website:** www.istructe.org

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UNIT 516: MECHANICS OF SOLIDS

Lesson 11: Determination of beam reactions	**Suggested Teaching Time:** 1 hour approx.

Learning Outcome 1: Be able to determine reactions and forces, bending moments shear forces and deflections

Topic	Suggested Teaching	Suggested Resources
Determination of beam reactions - revision	• Principle of moments. The basic principles should be demonstrated using simple equipment, and learners should be given the opportunity to test the principles practically. Whole-class teaching should reinforce the practical work to embed the concept that a moment of a force is the product of the magnitude of that force and the **perpendicular** distance between the turning point and the **line** of action of the force. Tutors must stress that it is **not** the actual distance between the **point** at which the force acts and the turning point that matters and, as above, this should be demonstrated practically. Learners should then be introduced to the 'principle of moments' – that clockwise moments and anti-clockwise moments are equal for co-planar systems in equilibrium. ($\Sigma M = 0$). This too should be demonstrated practically and/or checked practically by learners in small groups. • Total load = total reaction. A tutor-led discussion should be used to extend the concept of static equilibrium to the realisation that, for systems in equilibrium, not only does $\Sigma M = 0$, but ΣV and ΣH also equal zero. That $\Sigma V = 0$ implies that the algebraic sum of all vertical forces equals zero, and that total loads equal total reactions in all cases. The discussion should then be directed towards an understanding of how this concept, together with the principle of moments, can be used to determine the value of reactions for loaded beams.	**Books:** Hulse, R., Cain, J., Structural Mechanics ISBN: 0333804570 Hulse, R., Cain, J., Structural Mechanics (Worked Examples) ISBN: 0230579817 **Practical equipment** Beams, rules, hanging weights, pulleys, string **Software:** Goya Siemens PLM RISA Technology **Website:** www.istructe.org

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UNIT 516: MECHANICS OF SOLIDS

Lesson 12: Composite Beams		**Suggested Teaching Time:** 6 hours approx.
Learning Outcome 2: Understand the behaviour of elastically loaded structures		
Topic	**Suggested Teaching**	**Suggested Resources**

Composite Beams	• Continuing from the previous lesson, hold a Tutor-led discussion to develop the idea of using composite beams instead of solid metal or wooden ones. It must be emphasised that the materials are firmly bonded so that the strain at the interface is the same for both materials.
	• Whole-class teaching to develop the formulas from steel beams to those of composite beams. To include:
	  ○ calculating the second moment of area of composite beam cross-sections
	  ○ selecting values of second moment of areas for standard beams
	  ○ calculating the stress of a beam with built-in support
	  ○ calculating the deflection of a beam with built-in support
	  ○ calculating stresses of a composite beam subject to different loading conditions
	    ▪ These to include: UDL, point loads, combination
	  ○ calculating deflection of a composite beam subject to different loading conditions
	Students then to be given the opportunity to solve a variety of equations involving composite beams. Tutor to assist individual students, correcting errors as required. Where possible include practical elements.
		Books:
		Hibbeler, R. C. (2004). Statics and Mechanics of Materials (Second ed.). Pearson Prentice Hall. ISBN 0-13-028127-1.
		Software:
		computer-based finite element analysis software
		E.g. Autodesk http://www.autodesk.com/education/homeWebsite:
		http://www.freestudy.co.uk/mech%20prin%20h2/moments%20of%20area.pdf
		http://www.freestudy.co.uk/statics/beams/beam%20tut1.pdf
		http://mech.vub.ac.be/teaching/info/Ontwerpmethodologie/Appendix%20les%203%20Useful%20Problems.pdf

UNIT 516: MECHANICS OF SOLIDS

Lesson 13: Composite beams in frameworks		**Suggested Teaching Time: 4** hours approx.
Learning Outcome 2: understand the behaviour of elastically loaded structures		
Topic	**Suggested Teaching**	**Suggested Resources**
Determination of magnitude and type of forces in frameworks	• Whole-class teaching to build on previous knowledge of steel beams in structures to develop it into composite beam use in structures. To include: ○ Bow's notation. The tutor should develop the system used to annotate frames and the learners should then have the opportunity to annotate a series of different frames. This should be checked by the tutor. ○ Grap cal method of solving frames. This can be done manually or electronically. Whichever method is used, the tutor must stress the importance of accuracy in the drawing of both angles and lines. A discussion should follow in which the learners learn to differentiate between struts and ties from the direction of the forces in the individual force polygons joint. ○ Method of resolution. The tutor should demonstrate solving frames using horizontal and vertical static equilibrium at each joint. Learners should then practice on different frames with different loadings. ○ Method of sections. The tutor should demonstrate the procedures to use. A class discussion should ensue on where and when this method might be preferred to the other methods available, given that it is generally perceived to be more difficult. ▪ Examples to include: Roof truss, cantilever, redundant member. Tutor to provide students with example composite frameworks (physical examples if possible) for the students to analyse using knowledge learned so far.	**Books:** Hulse, R., Cain, J., Structural Mechanics ISBN: 0333804570 Durka F et al Structural Mechanics ASBN: 0132239647 **Practical equipment** Proprietary equipment for testing frames **Software:** Goya RISA Technology **Website:** www.istructe.org

UNIT 516: MECHANICS OF SOLIDS

Lesson 14: Fatigue		**Suggested Teaching Time: 4** hours approx.
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**

Fatigue
(A.C. 3.1)
	• Whole-class discussion to cover the concept of how to apply the theories that have been learned so far and to discuss how life is not always as simple as a single load being placed on a beam and reduced once. Introduce the concept of cyclic loading.
	• A simple demonstration is the use of a plastic rule bent backwards and forwards till it snaps.
	• Tutor to demonstrate the setting up a cyclic loading experiment and then issue the students with a suitable selection of samples of materials and get the students to set up their own experiments. Students to carry out their experiments and to record the results.
	• The tutor should then demonstrate how a stress to Cycles graph (S-N curve) is drawn and then get students to produce one for each of the materials they have tested.
	• Tutor can then discuss the results with the class and compare the failure stress with the expected failure stress of the material under tensile testing.
	• Tutor to get the students to plot an (S-N curve).
	    ○ analyse an engineering component subject to **fatigue loading including:**
	        ▪ Life factors, S-N curves
	• Whole-class teaching to cover: Miners rule Paris' Law and the Goodman relation.
	• Whole-class discussion on the concept of low cycle fatigue and to bring in the Coffin-Manson relation.
	
	Split class into smaller groups and issue a series of questions covering the equations used so far.
	
	Where possible, include practical elements. Tutor to circulate and correct as required.
		Books:
		Comprehensive Structural Integrity: Cyclic loading and fatigue
		edited by I. Milne, Robert O. Ritchie, B. L. Karihaloo
		Publisher Elsevier, 2003
		ISBN: 0080441556, 9780080441559
		Practical equipment:
		Proprietary equipment for conducting cyclic loading experiments
		Software:
		Goya
		RISA Technology
		computer-based finite element analysis software
		E.g. Autodesk
		http://www.autodesk.com/education/homeWebsite:
		Website:
		http://materials.open.ac.uk/mem/mem_mf1.htm

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UNIT 516: MECHANICS OF SOLIDS

Lesson 15: Creep	**Suggested Teaching Time: 4** hours approx.

Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions

Topic	Suggested Teaching	Suggested Resources

Creep
 	
(A.C. 3.2)
	• This lesson requires a significant timescale to conduct the experiments, so it is worthwhile considering setting up the experiment the week previous to this, letting it run, and getting the students to analyse the data during this lesson.
	• Whole-class discussion to introduce the concept of creep, how materials under constant stress will try and change shape, and how this must be limited. A good example is that of a turbine blade in a gas turbine engine, spinning at several thousand RPM at temperatures of over 1000 degrees.
	• Students to be shown how to set up the testing and recording equipment to conduct creep testing of materials. This should include materials at different temperatures for comparison. They are to be given a selection of materials to test and then tasked with setting up and conducting the experiments.
	• Analyse an engineering component involving **creep** to include:
	    ▪ elevated temperature, stress rupture curves
	    ▪ use stress rupture curve for selected material.
	
	Whole-class teaching to cover creep mechanisms, dislocation slip and climb Grain boundary sliding and diffusion flow.
		Books:
		Hulse, R., Cain, J.,
		Structural Mechanics
		ISBN: 0333804570
		Durka F et al
		Structural Mechanics
		ASBN: 0132239647
		The Science & Engineering of Materials
		Donald Askeland, Pradeep Fulay
		Cengage Learning, 2005
		ISBN 0534553966, 9780534553968
		Practical equipment:
		Proprietary equipment for testing materials subject to creep
		Website:
		http://www.doitpoms.ac.uk/tlplib/creep/simulation.php

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Lesson 16: Viscoelastic materials	**Suggested Teaching Time: 6** hours approx.	
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**
Viscoelastic materials (A.C. 3.3)	• Tutor-led discussion to cover viscoelasticity; viscoelasticity is not plasticity, with which it is often confused. A viscoelastic material will return to its original shape after any deforming force has been removed (i.e., it will show an elastic response) even though it will take time to do so (i.e., it will have a viscous component to the response). • Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain when stretched and quickly return to their original state once the stress is removed. Viscoelastic materials have elements of both of these properties and, as such, exhibit time-dependent strain. • Linear viscoelasticity is when the function is separable in both creep response and load. All linear viscoelastic models can be represented by a Volterra equation connecting stress and strain: $\varepsilon(t) = \frac{\sigma(t)}{E_{inst,creep}} + \int_0^t K(t-t')\dot{\sigma}(t')dt'$ or: $\sigma(t) = E_{inst,realx}\varepsilon(t) + \int_0^t F(t-t')^{\epsilon(\dot{t'})}dt'$ where: t is time σ(t) is stress ε (t) is strain $E_{inst,creep}$ and $E_{inst,relax}$ are instantaneous elastic moduli for creep and relaxation K(t) is the creep function F(t) is the relaxation function. Linear viscoelasticity is usually applicable only for small deformations.	**Books:** Viscoelastic Materials, Roderick S. Lakes Cambridge University Press, 2009 ISBN 052188568X, 9780521885683 **Practical equipment:** Viscoelastic shapes and laboratory analysis equipment for stress strain and indention **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://engineeringexploration.autodesk.com/content/module-4-viscoelastic-materials **Video** http://www.youtube.com/watch?v=Q1VtAXeMn74

Topic	Suggested Teaching	Suggested Resources
Viscoelastic materials (Continued) (A.C. 3.3)	• Nonlinear viscoelasticity is when the function is not separable. It usually happens when the deformations are large or if the material changes its properties under deformations. • An anelastic material is a special case of a viscoelastic material: an anelastic material will fully recover to its original state on the removal of load. • Tutor to explain how viscoelastic behaviour has elastic and viscous components modelled as linear combinations of springs and dashpots, The elastic components, as previously mentioned, can be modelled as springs of elastic constant E, given the formula: $\sigma = E\varepsilon$ • Where σ is the stress, E is the elastic modulus of the material, and ε is the strain that occurs under the given stress, similar to Hooke's Law. • The viscous components can be modelled as dashpots such that the stress–strain rate relationship can be given as: where σ is the stress, η is the viscosity of the material, and dε/dt is the time derivative of strain. $\sigma = \eta \frac{d\varepsilon}{dt}$ Tutor to develop these formulae into the Maxwell model, the Kelvin–Voigt model, and the Standard Linear Solid Model and to show how they are used to predict a materials response under different loading conditions.	**Books:** Viscoelastic Materials, Roderick S. Lakes Cambridge University Press, 2009 ISBN 052188568X, 9780521885683 **Practical equipment:** Viscoelastic shapes and laboratory analysis equipment for stress strain and indention **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://engineeringexploration.autodesk.com/content/module-4-viscoelastic-materials **Video:** http://www.youtube.com/watch?v=Q1VtAXeMn74
Revision of Viscoelastic materials (A.C. 3.3)	• Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required.	

UNIT 516: MECHANICS OF SOLIDS

Lesson 17: Failure of materials		**Suggested Teaching Time: 4** hours approx.
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**
Failure of materials (A.C. 3.4 & 3.5) Linear Elastic Fracture Mechanics (LEFM) (A.C. 3.4 & 3.5)	○ Whole-class discussion about what has been covered in the subject so far, referencing stress, strain, creep and fatigue. ○ Tutor to give examples of calculations used so far and get the students to complete them as a revision exercise. Tutor to assist individual students, correcting errors as required. ○ Tutor to lead a discussion covering the fact that these calculations are based on standard 'perfect' samples and to get the students to come up with factors that will affect these results. Try to get answers on design flaws, as well as internal flaws, e.g. comet airliner with square windows that cracked. Tutor to include the topic of stress intensity factor. ○ Linear Elastic Fracture Mechanics (LEFM) first assumes that the material is isotropic and linear elastic. Based on this assumption, the stress field near the crack tip is calculated using the theory of elasticity. When the stresses near the crack tip exceed the material fracture toughness, the crack will grow. ○ In Linear Elastic Fracture Mechanics, most formulas are derived for either plane stresses or plane strains, associated with the three basic modes of loadings on a cracked body: opening, sliding, and tearing. LEFM is valid only when the inelastic deformation is small compared to the size of the crack, small-scale yielding. If large zones of plastic deformation develop before the crack grows, Elastic Plastic Fracture Mechanics (EPFM) must be used.	**Books:** Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention Jack A. Collins John Wiley & Sons, ISBN 0471558915, 9780471558910 **Practical equipment:** **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsiteWebsite: http://www2.mae.ufl.edu/haftka/adv-elast/lectures/Chapter15.1-2.pdf

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Topic	Suggested Teaching	Suggested Resources
Linear Elastic Fracture Mechanics (LEFM) (continued) (A.C. 3.4 & 3.5)	○ LEFM analysis can be outlined as follows: ○ Based on linear elasticity theories, the stress field near a crack tip is a function of the location, the loading conditions, and the geometry of the specimen or object. $\sigma_{ij}^{Tip} \equiv \sigma_{ij}^{Tip}(Location, Loading, Geometry)$ ○ In practice, engineers calculate the stress intensity factor K based on the stress field at the crack tip and compare it against the known fracture toughness of the material: The crack tip stress field is a function of the location, loading, and geometry: $\sigma_{ij}^{Tip} \equiv \sigma_{ij}^{Tip}(Location, Loading, Geometry)$ $\equiv \sigma_{ij}^{Tip}(r, \theta, K)$ ○ Where location can be represented by r and θ using the polar coordinate system whereas the loading and geometry terms can be grouped into a single parameter K, called the stress intensity factor. $K \equiv K\left(\sigma^{Loading}, Geometry\right)$ ○ The stress intensity factor K for a few simple loading and geometry conditions are grouped in three categories: classic, specimen, and structure. The fracture toughness of a material can be obtained by experiment. It is material specific. $\sigma_{ij}^{Toughness} \equiv \sigma_{ij}^{Toughness}(Material)$ ○ The stress intensity factor associated with the fracture toughness of the material is called the critical stress intensity factor Kc where Kc is material dependent. $K_c \equiv K_c(Material)$ Tutor to provide samples of materials that fail in this method and get students to carry out failure analysis on them.	**Books:** Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention Jack A. Collins John Wiley & Sons, ISBN 0471558915, 9780471558910 **Practical equipment:** **Software:** Software: computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://www2.mae.ufl.edu/haftka/adv-elast/lectures/Chapter15.1-2.pdf http://www.eng.fsu.edu/~chandra/courses/eml3004c/Fracturemechanics-ppt/ch5-EPFM.ppt

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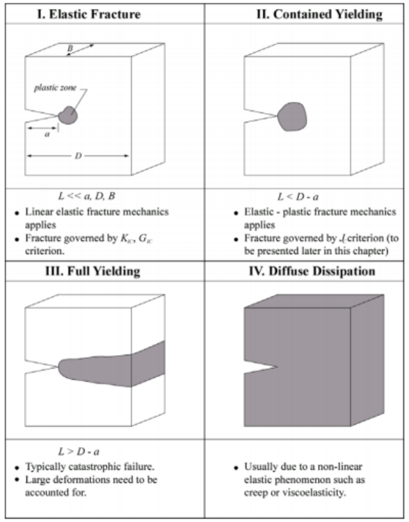
Lesson 18: Failure of materials		**Suggested Teaching Time:** 6 hours approx.
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**
Approximating the size of the plastic zone (A.C. 3.4 & 3.5)	○ Tutor-led discussion on how up until now it has been assumed that the size of the plastic zone near the crack tip is relatively small as compared to the specimen dimensions. Thus, the effect of this zone has been neglected and the strain field surrounding the crack tip is dominated by linear elastic fracture mechanics asymptotic field. In this lesson the effects of a non-negligible plastic zone is considered in the case of Mode I only. First, in order to determine at what point it is necessary to include the influence of the plastic zone in the stress analysis. ○ Discuss the two methods of approximation for the size of the plastic zone (one dimensional estimation including von Mises yield criterion) in each of four cases. 	**Books:** Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention Jack A. Collins John Wiley & Sons, ISBN 0471558915, 9780471558910 **Practical equipment:** Materials laboratory **Software:** Software: computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://www2.mae.ufl.edu/haftka/adv-elast/lectures/Chapter15.1-2.pdf http://lmafsrv1.epfl.ch/Laurent/Fracture%20Mechanics/CH_6-%20Elastic-Plastic%20Fracture%20Mechanics.pdf

Topic	Suggested Teaching	Suggested Resources
Approximating the size of the plastic zone (Continued) (A.C. 3.4 & 3.5)	○ Two dimensional approximation (2D). ○ While the length L gives us an indication of the size of the plastic zone relative to the specimen dimensions, it is also useful to know the actual shape of the plastic zone around the crack tip. Using the same approach as in the previous section, it is not difficult to obtain its shape. ○ In cylindrical coordinates the principle stresses 1 σ and 2 σ for an arbitrary stress field are given by: $\sigma_{,2} = \left(\frac{\sigma_{\theta\theta} + \sigma_{lt}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{\theta\theta} + \sigma_{lt}}{2}\right)^2 + {\sigma^2}_{r\theta}}$ Near the crack tip, we obtain after some algebraic manipulation: $\sigma_{1,2} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left(1 \pm \sin\frac{\theta}{2}\right)$ and $\sigma_3 = v(\sigma_1 + \sigma_2)$ in the case of plane strain. ○ Tutor to demonstrate how these formula can be manipulated to solve the radius of the plastic zone by the formula: $r_P(\theta) = \frac{1}{2\pi}\frac{K_I^2}{r} cos^2\left(\frac{\theta}{2}\right)\left[4(1 - v + v^2) - 3cos^2\left(\frac{\theta}{2}\right)\right]$ ○ The shape of the plastic zone should then be plotted for v = 1/3 and = 1/2. And the limiting case of plane stress (i.e. v = 0) should also be plotted. ○ This will show how the state of stress changes through the thickness of a thick specimen (i.e. plane stress → plane strain), the shape of the plastic zone also changes. ○ Note that the shapes of the plastic zone shown do not take into account the effects of the specimen boundaries. Thus, they are derived for a crack in an infinite plate. For an actual specimen, one must often consider the finite width in order to determine the region of plastic deformation. ○ Whole of class teaching to cover alternative methods of approximation, to include: Irwin's approximation of the plastic zone and Dudgale's yield strip model. ○ Whole-class teaching to cover Crack Tip Opening Displacement as Yield Criterion. ○ For linear elastic fracture mechanics, tutor to discuss how we have developed yield criterion such as K and G and how we must now develop similar yield criterion for the case II, contained yielding.	**Books:** Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention Jack A. Collins John Wiley & Sons, ISBN 0471558915, 9780471558910 **Practical equipment:** Materials laboratory **Software:** Software: computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://www2.mae.ufl.edu/haftka/adv-elast/lectures/Chapter15.1-2.pdf http://lmafsrv1.epfl.ch/Laurent/Fracture%20Mechanics/CH_6-%20Elastic-Plastic%20Fracture%20Mechanics.pdf

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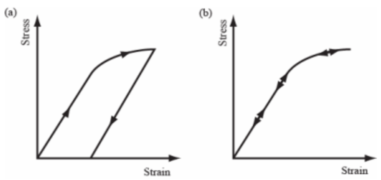
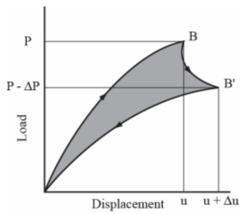
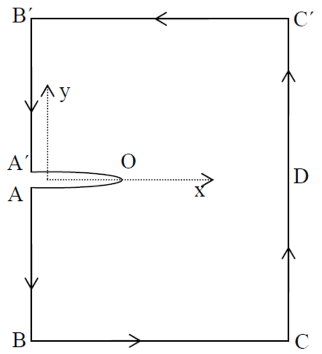
Lesson 19: Failure of materials	**Suggested Teaching Time: 6** hours approx.	
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**
The J integral as yield criterion - Deformation theory of plasticity (A.C. 3.4 & 3.5)	○ Whole-class teaching to derive a fracture criterion for an elastic-plastic material which has a stress-strain behaviour as shown below:  Tutor to discuss how for small loads, the stress-strain curve is linear. But beyond a critical load, the curve is non-linear and as the load is released, the stress-strain curve is once again linear, with the same slope as during the initial loading. Tutor to explain that the major difficulty in modelling this material is that the behaviour is no longer reversible (i.e. the stress-strain curve does not follow the same path for loading and unloading). Thus, the material behaviour is now dependent upon the strain history and the stress at a given strain is no longer unique. However, this effect does not become important unless unloading occurs. They should therefore emphasise that, for the derivations that follow, the elastic-plastic behaviour will be replaced by the non-linear case, and specify that they consider only monotonic (non-cyclic) loading. This substitution is called the Deformation Theory of Plasticity. Note that this substitution is not always valid for a truly 3D case of loading; however it is valid for several cases. Tutor to define what is J  	**Books:** Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention Jack A. Collins John Wiley & Sons, ISBN 0471558915, 9780471558910 **Practical equipment:** Materials laboratory **Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: **Website:** http://www2.mae.ufl.edu/haftka/adv-elast/lectures/Chapter15.1-2.pdf http://lmafsrv1.epfl.ch/Laurent/Fracture%20Mechanics/CH_6-%20Elastic-Plastic%20Fracture%20Mechanics.pdf

	○ Load-displacement curve of non-linear elastic material. The shaded area gives energy, released from state B to state B', defined as J. Tutor to discuss the properties of J. ○ The J-integral along a specific contour to illustrate the use of the J-integral, the tutor should use a similar example to this one. ○ A cracked specimen loaded in Mode I and crack free surfaces. Without the need to know the form of the loading, they can show the students how to proceed to determine the explicit forms of the J-integral along a symmetric contour ABCDC´B´A´ shown below:  ○ The contour shown can be either along the border of the specimen or the contour around the crack in a larger specimen or component. The crack faces A´O and AO are traction free. It is also understood that the chosen contour can be chosen along the boundaries of the specimen or in its interior. The tutor should then go on to show how this can be solved. The tutor should demonstrate how a simple yet important relationship can be derived between the CTOD and the J-integral for the yield strip model.	

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Lesson 20: Revision and computer modelling Part 3	**Suggested Teaching Time: 6** hours approx.	
Learning Outcome 3: Understand the nature of failure modes under plastically loaded conditions		
Topic	**Suggested Teaching**	**Suggested Resources**
Revision of Topic	Whole-class discussion to revise whole topic, including formulas used. Tutor to demonstrate how these calculations can be solved using finite analysis software. Split class into smaller groups and issue a series of questions covering the equations used so far, which the students should solve manually and by using the software. Where possible physical examples should be used which can also be tested using measuring equipment. Tutor to circulate and correct as required.	**Software:** computer-based finite element analysis software E.g. Autodesk http://www.autodesk.com/education/homeWebsite: