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City & Guilds

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Level 5 Advanced Technician Diploma in Electrical and Electronic Engineering
Level 5 Advanced Technician Diploma in Mechanical Engineering

Instrumentation and control systems

Tuesday 24 November 2015
09:30 – 12:30

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Laplace Transforms

If $y(t)$ is a function defined for $t \geq 0$, the Laplace transform $\overline{y}(s)$ is defined by the equation

$$\overline{y}(s) = \mathcal{L}\{y(t)\} = \int_0^\infty \mathrm{e}^{-st} y(t)\, \mathrm{d}t$$

Function $y(t)$ $(t > 0)$	Transform $\overline{y}(s)$	
$\delta(t)$	1	Delta function
$\theta(t)$	$\dfrac{1}{s}$	Unit step function
t^n	$\dfrac{n!}{s^{n+1}}$	
$t^{1/2}$	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{s^3}}$	
$t^{-1/2}$	$\sqrt{\dfrac{\pi}{s}}$	
e^{-at}	$\dfrac{1}{(s+a)}$	
$\sin \omega t$	$\dfrac{\omega}{(s^2+\omega^2}$	
$\cos \omega t$	$\dfrac{s}{(s^2+\omega^2)}$	
$\sinh \omega t$	$\dfrac{\omega}{(s^2-\omega^2)}$	
$\cosh \omega t$	$\dfrac{s}{(s^2-\omega^2)}$	
$\mathrm{e}^{-at} y(t)$	$\overline{y}(s+a)$	
$y(t-\tau)\, \theta(t-\tau)$	$\mathrm{e}^{-s\tau}\, \overline{y}(s)$	
$ty(t)$	$-\dfrac{\mathrm{d}\overline{y}}{\mathrm{d}s}$	
$\dfrac{\mathrm{d}y}{\mathrm{d}t}$	$s\overline{y}(s) - y(0)$	
$\dfrac{\mathrm{d}^n y}{\mathrm{d}t^n}$	$s^n \overline{y}(s) - s^{n-1} y(0) - s^{n-2}\left[\dfrac{\mathrm{d}y}{\mathrm{d}t}\right]_0 \cdots - \left[\dfrac{\mathrm{d}^{n-1} y}{\mathrm{d}t^{n-1}}\right]_0$	
$\displaystyle\int_0^t y(\tau)\, \mathrm{d}\tau$	$\dfrac{\overline{y}(s)}{s}$	
$\displaystyle\int_0^t x(\tau)\, y(t-\tau)\, \mathrm{d}\tau$ $\displaystyle\int_0^t x(t-\tau)\, y(\tau)\, \mathrm{d}\tau$	$\overline{x}(s)\, \overline{y}(s)$	Convolution theorem

[Note that if $y(t) = 0$ for $t < 0$ then the Fourier transform of $y(t)$ is $\hat{y}(\omega) = \overline{y}(\mathrm{i}\omega)$.]

The online version of the full Mathematical handbook can be found at
http://homepage.ntu.edu.tw/~wttsai/MathModel/Mathematical%20Formula%20Handbook.pdf