

Candidates should refer to this 505 source document throughout the course and become familiar with it. Candidates will need to use a clean copy of this document for the 9205-505 exam and for the sample questions.

The online version of the full Mathematical handbook can be found at <http://homepage.ntu.edu.tw/~wtttsai/MathModel/Mathematical%20Formula%20Handbook.pdf>

(Please note we have extracted formula which is relevant to this unit. The full online version is not required.)

Laplace Transforms

If $y(t)$ is a function defined for $t \geq 0$, the Laplace transform $\bar{y}(s)$ is defined by the equation

$$\bar{y}(s) = \mathcal{L}\{y(t)\} = \int_0^{\infty} e^{-st} y(t) dt$$

Function $y(t)$ ($t > 0$)	Transform $\bar{y}(s)$	
$\delta(t)$	1	Delta function
$\theta(t)$	$\frac{1}{s}$	Unit step function
t^n	$\frac{n!}{s^{n+1}}$	
$t^{1/2}$	$\frac{1}{2} \sqrt{\frac{\pi}{s^3}}$	
$t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$	
e^{-at}	$\frac{1}{(s+a)}$	
$\sin \omega t$	$\frac{\omega}{(s^2 + \omega^2)}$	
$\cos \omega t$	$\frac{s}{(s^2 + \omega^2)}$	
$\sinh \omega t$	$\frac{\omega}{(s^2 - \omega^2)}$	
$\cosh \omega t$	$\frac{s}{(s^2 - \omega^2)}$	
$e^{-at} y(t)$	$\bar{y}(s+a)$	
$y(t-\tau) \theta(t-\tau)$	$e^{-s\tau} \bar{y}(s)$	
$ty(t)$	$-\frac{d\bar{y}}{ds}$	
$\frac{dy}{dt}$	$s\bar{y}(s) - y(0)$	
$\frac{d^n y}{dt^n}$	$s^n \bar{y}(s) - s^{n-1} y(0) - s^{n-2} \left[\frac{dy}{dt} \right]_0 - \dots - \left[\frac{d^{n-1} y}{dt^{n-1}} \right]_0$	
$\int_0^t y(\tau) d\tau$	$\frac{\bar{y}(s)}{s}$	
$\left. \begin{aligned} &\int_0^t x(\tau) y(t-\tau) d\tau \\ &\int_0^t x(t-\tau) y(\tau) d\tau \end{aligned} \right\}$	$\bar{x}(s) \bar{y}(s)$	Convolution theorem

[Note that if $y(t) = 0$ for $t < 0$ then the Fourier transform of $y(t)$ is $\hat{y}(\omega) = \bar{y}(i\omega)$.]