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Level 5 Advanced Technician Diploma in Mechanical Engineering
Advanced Engineering Mathematics

Monday 16 November 2015
09:30 – 12:30

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Mathematical Formulae Sheet

Taylor series expansion of *f*(a + x):

$$f(a+x) = f(a) + \frac{x}{1!}f^{(1)}(a) + \frac{x^2}{2!}f^{(2)}(a) + \frac{x^3}{3!}f^{(3)}(a) + \cdots$$

where x is the displacement measured from the fixed point a

where $f^{(n)}(a)$ = n'th derivative of *f*(x) evaluated at *x* = *a*.

Maclaurin series expansion of *f*(x):

This has the same expansion as for the Taylor series but with a = 0 thus,

$$f(x) = f(0) + \frac{x}{1!}f^{(1)}(0) + \frac{x^2}{2!}f^{(2)}(0) + \frac{x^3}{3!}f^{(3)}(0) + \cdots$$

Fourier series description of *f*(x):

(a) for functions with period 2π

$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$, where

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \cos nx\, dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \sin nx\, dx$

(b) for functions *f*(*t*) with period *T* in seconds

i.e. frequency in hertz $f_h = \frac{1}{T}$ or angular frequency $\omega = \frac{2\pi}{T}$

$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\ \cos n\omega t + b_n\ \sin n\omega t$, where

$a_0 = \frac{2}{T}\int_0^T f(t)\, dt$

$a_n = \frac{2}{T}\int_0^T f(t)\ \cos n\omega t\, dt$

$b_n = \frac{2}{T}\int_0^T f(t)\ \sin n\omega t\, dt$

Trapezoidal Rule using *n* subintervals of the interval [*a*,*b*] each of width *h*:

$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(a) + f(b) + 2\sum_{k=1}^{n-1} f(a + kh)]$

Simpson's Rule with even number (n) of subintervals for [*a,b*], each of width *h*:

$\int_a^b f(x)\,dx \approx \frac{h}{3}[f(a) + f(b) + 2\sum_{r=1}^{n-1} f(a + 2rh) + 4\sum_{r=1}^{n} f(a + \{2r - 1\}h)]$

Euler numerical method for the solution of $\frac{dy}{dx} = f(x,y)$ using a step size *h*:

$y_{n+1} = y_n + h\,f(x_n, y_n)$

Improved Euler numerical method:

$y_{n+1}^0 = y_n + h\,f(x_n, y_n)$ then

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f^0(x_{n+1}, y_{n+1}^0)]$